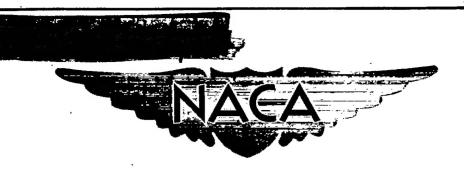
Сору

203

RM A52J22

NACA RM A52722

6395





RESEARCH MEMORANDUM

THEORETICAL INVESTIGATION OF THE PERFORMANCE OF PROPORTIONAL

NAVIGATION GUIDANCE SYSTEMS - EFFECT OF MISSILE

CONFIGURATION ON THE SPEED OF RESPONSE

By Marvin Abramovitz

Ames Aeronautical Laboratory
Moffett Field, Calif.





NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

January 19, 1953



NACA RM A52J22





NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

THEORETICAL INVESTIGATION OF THE PERFORMANCE OF PROPORTIONAL

NAVIGATION GUIDANCE SYSTEMS - EFFECT OF MISSILE

CONFIGURATION ON THE SPEED OF RESPONSE

By Marvin Abramovitz

SUMMARY

A comparison of the maximum speed of response that can be attained by three missile configurations, a variable-incidence-wing, a canard, and a tail-control, in combination with a particular proportional navigation guidance system is presented. It was found that the configuration which allows the most rapid over-all guidance-system response depends on the control-system characteristics. With rate feedback only in the missile stabilization system the variable-incidence-wing missile resulted in a slightly faster response, with an oscillation frequency very nearly that of the missile alone. With normal-acceleration feedback also included, a more rapid response was attained with the tail-control missile. The frequency for all three configurations with acceleration feedback was higher than that of the missiles alone, but objectionable variations of navigation ratio with missile-flight-speed changes were present.

INTRODUCTION

Among the antiaircraft homing missiles being designed and developed, examples of all three basic configurations investigated in this report, a variable-incidence-wing, a canard, and a tail-control, can be found. The choice of the optimum configuration depends on a number of factors, some of which are: the speed of response, weight, drag, servo energy requirements, convenient location of control-system components, and size limitations. The present investigation is concerned with the first of these factors, the speed of response.

A variable-incidence-wing configuration can be designed to have a more rapid response to a control-surface deflection than either a canard



or a tail-control missile having the same natural frequency. This is due to the immediate build-up of a large portion of the lift on the wing before the missile pitches to the trim angle of attack. When the response of a complete seeker, control-system, and missile combination is considered, some of the advantage of the variable-incidence configuration can be lost due to the delay in surface deflection, which results from the effects of lags in the various components of the system. In addition, because of the large control-surface area of the variable-incidence-wing configuration, a weight penalty is incurred due to the greater servo energy requirements. These factors were investigated in reference 1 at a design flight condition for a beam-rider guidance system.

In the present investigation, the speeds of response attainable with the three missile configurations in combination with a proportional navigation guidance system are compared. The type of guidance system, shown in figure 1 in block diagram form, is one which previous investigation has shown to have desirable speed of response characteristics (reference 2). In this system the antenna is stabilized in space and the complete system can be separated into two distinct parts: the seeker, which, with high gearings, introduces only small lags into the over-all response; and the missile-control-system combination, the dynamic characteristics of which determine to a large extent the over-all system speed of response.

The control system utilizes both rate of pitch and normal acceleration feedback. For the first portion of the investigation, the acceleration feedback gearing is zero so that only rate feedback is present and the control system is identical to the rate feedback control system of reference 2. For the remaining portion of the investigation, both rate and acceleration feedback are used.

The missiles are assumed to be of the boost-glide type and the effect of the decreasing flight speed during the unpowered portion of the homing trajectory is investigated by considering two Mach numbers: one that corresponds to the nominal design condition at the end of boost, and a second lower speed to correspond to conditions near the end of controlled flight. In addition, the effect of low static margin is investigated.

In this investigation, as in references 1 and 2, the effects of radar noise are neglected. Simplified, noiseless trajectory studies (reference 3) show that the magnitude of the miss distance is directly proportional to the guidance-system lag. Therefore, in the absence of noise, it is presumed that the selection of the most desirable missile configuration will depend on the speed of response attainable, provided that adequate system stability is present.



WE TOTAL I THE

NOTATION

$$c_L \frac{L}{qS_b}$$

$$C_m = \frac{M}{qS_bd}$$

c,C transfer-function coefficients

D,(') time derivative
$$\left[\frac{d()}{dt}\right]$$

d body diameter, feet

Iy moment of inertia in pitch, slug-feet squared

$$i_y = \frac{I_y}{qS_{bd}}$$
, seconds squared

K gearing (static gain)

distance between center of pressure due to angle of attack and center of pressure due to control deflection, feet

L lift, pounds

M pitching moment, pound-feet (or Mach number)

m mass, slugs

N navigation ratio $\left(\frac{\mathring{7}}{\mathring{6}}\right)_{\text{steady state}}$

n $\left(\frac{V}{V_c}\right)$ N, approximately

p a variable introduced in the Laplace transformation

q dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$

 $\mathbf{S}_{\mathbf{b}}$ maximum missile body cross-sectional area, feet squared

Sf exposed area of two front surface panels, feet squared

COMPONETAL

- Sr exposed area of two rear surface panels, feet squared
- T_{θ} ! lead term of missile $\left(\frac{\dot{\theta}}{\delta}\right)$ transfer function for X=0, seconds
- t transfer-function lead term (or time), seconds
- V missile flight speed, feet per second
- ${\tt V_c}$ missile-target closing speed, feet per second
- $V_{\tau p}$ target flight speed, feet per second
- v voltage
- X static margin parameter $\left(\frac{x}{d} \frac{c_m \delta}{\tau}\right)$
- x distance between center of gravity and center of pressure due to angle of attack, feet
- y distance between center of gravity and center of pressure due to control deflection, feet
- angle of attack, radians
- γ flight-path angle, radians
- δ control deflection, radians
- ε radar antenna error angle, radians
- θ angle of pitch, radians
- λ angular orientation of radar antenna axis with respect to missile longitudinal axis, radians
- ρ air density, slugs per cubic foot
- σ line of sight angle in space, radians
- $\tau \frac{2m}{\rho V S_{b}}$, seconds

Subscripts

- A antenna gyro precessing mechanism
- a normal accelerometer
- G rate gyro in control system
- m missile-control-system-combination transfer function

$$\left(rac{\dot{ heta}}{v_{
m R}}
ight)$$
 or $\left(rac{\dot{ textstyle y}}{v_{
m R}}
ight)$

- R radar receiver
- S control servo
- α <u>9α</u>
- ç <u>9¢</u>
- $\delta = \frac{96}{9()}$
- θ missile transfer function $\left(\frac{\dot{\theta}}{\delta}\right)$
- e <u>96</u>
- γ missile transfer function $\left(\frac{\dot{\gamma}}{\dot{\theta}}\right)$

DESCRIPTION OF MISSILES, CONTROL SYSTEM, AND GUIDANCE SYSTEM

Missiles

Figure 2 is a sketch of the three missile configurations, a variable-incidence-wing, a canard, and a conventional tail control with a cruciform arrangement of the triangular surfaces. These configurations are identical to those of reference 1 and are designed to have identical maneuvering capabilities and natural frequencies at an altitude of 50,000 feet and a Mach number of 2.7. The design conditions and the method of determining the aerodynamic characteristics at M=2.7 are described in reference 1.



Similar methods were used in determining the characteristics at M=1.3. The missile aerodynamic coefficients are based on the body diameter and body cross-sectional area for this report, since the body is identical for all three configurations while the wing and tail areas are different.

The transfer functions used in this report are of slightly different form than those of reference 1 or 2. This altered form is preferable for investigating the variation of static margin since, in the usual notation, several of the transfer-function coefficients become infinite near zero static margin. The transfer functions were determined from the usual simplified equations of motion

$$(I_{CC} + mVD)\alpha - mVD\theta = -I_{\delta}\delta$$

$$(-M_{CC} - M_{CC}^*D)\alpha + (-M_{C}^* + I_{y}D)D\theta = M_{\delta}\delta$$

$$-\alpha + \theta = \gamma$$

$$(1)$$

To determine the effects of static margin, it was assumed that the moment of inertia, I_y , and the damping derivatives, $C_{m_0^*}$ and $C_{m_0^*}$, remain essentially constant with variations in static margin. Furthermore, the following equations give the linear variation of the moment derivatives C_{m_0} and C_{m_0} with static margin:

$$C_{m_{CL}} = -(x/d)C_{L_{CL}}$$

$$C_{m_{CL}} = (y/d)C_{L_{CL}}$$

$$1/d = (x/d)+(y/d)$$
(2)

With the above relationships, the following transfer functions are obtained, in nondimensional form, from the equations of motion (equations (1)):

$$\frac{\dot{\theta}}{\delta} = \frac{K_{\theta}^* \left[1 + (T_{\theta}^* - c_{\theta}X)p\right]}{X + C_{\theta_1}p + C_{\theta_2}p^2}$$

$$\frac{\dot{\gamma}}{\dot{\theta}} = \frac{(1 + c_{\gamma_1}p + c_{\gamma_2}p^2)}{\left[1 + (T_{\theta}^* - c_{\theta}X)p\right]}$$
(3)

TA THE

where

$$\begin{split} & X = [(x/d) - (C_{m_{\theta}^{\bullet}}/\tau)] \\ & K_{\theta}! = (i/d) (C_{L_{0}}/\tau) \\ & T_{\theta}! = [(i/d) - (C_{m_{\theta}^{\bullet}}/\tau + C_{m_{\alpha}^{\bullet}}/\tau)]/[(i/d) (C_{L_{\alpha}}/\tau)] \\ & c_{\theta} = 1/[(i/d) (C_{L_{\alpha}}/\tau)] \\ & c_{\gamma_{1}} = -(C_{m_{\theta}^{\bullet}}/\tau + C_{m_{\alpha}^{\bullet}}/\tau)/[(i/d) (C_{L_{\alpha}}/\tau)] \\ & c_{\gamma_{2}} = (i_{y}/\tau)/[(i/d) (C_{L_{\alpha}}/\tau)] \\ & c_{\theta_{1}} = [(i_{y}/\tau) (C_{L_{\alpha}}/\tau) - (C_{m_{\theta}^{\bullet}}/\tau + C_{m_{\alpha}^{\bullet}}/\tau)]/(C_{L_{\alpha}}/\tau) \\ & c_{\theta_{2}} = (i_{y}/\tau)/(C_{L_{\alpha}}/\tau) \end{split}$$

Values of the mass and aerodynamic parameters for the three configurations at M=2.7 and M=1.3 and an altitude of 50,000 feet are tabulated in table I.

Control System

Figure 3 is a block diagram of the control system. In the steady state, the control system produces a missile rate of turn, $\dot{\gamma}$, proportional to the radar output voltage. The constant of proportionality (controlsystem gearing) depends on the individual gearings in the following manner:

$$(\dot{\gamma}/v_{\rm R})_{\rm S \bullet S \bullet} = K_{\rm m} = (K_{\rm S}K_{\rm \theta}^*/X)/(1 + K_{\rm S}K_{\rm G}K_{\rm \theta}^*/X + K_{\rm S}VK_{\rm B}K_{\rm \theta}^*/X)$$
 (4)

The rate feedback is necessary to provide increased damping since the missiles themselves are poorly damped, having damping ratios from 0.04 to 0.07 (depending on the configuration) for the normal design static margin and a Mach number of 2.7. In order to obtain adequate damping, it is necessary to include a rate gyro lead, t_G, approximately equal to the control servo lag.

With only rate feedback (i.e., with $K_a=0$), the control-system gearings can be selected so that the system oscillation frequency is approximately the same as the missile natural frequency. (See reference 2.) Since at low static margins the missile natural frequency is low, this



control system was only investigated at the normal design static margin where a rapid response could be expected. With normal acceleration feedback included, it is shown in reference 4 that a system oscillation frequency can be obtained that is higher than the missile natural frequency. Therefore, the investigation included a range of missile static margins for this control system and results are presented for the normal design static margin and for a very small negative static margin.

Guidance System

Figure 1 is a block diagram of the guidance system. This system is identical to system II of reference 2. In reference 2, it is shown that a rapid response can be obtained with this system in which the radar antenna is stabilized in space so that no coupling occurs between the antenna motion and the missile turning motion. With a high value for the seeker open-loop gearing, K_AK_R , the seeker responds with a voltage output proportional to the rate of rotation of the line of sight, $\mathring{\sigma}$, with negligible lag. The control system produces a missile rate of turn, $\mathring{\gamma}$, proportional to the seeker output in the steady state so that the complete guidance system produces proportional navigation in the steady state in compliance with the equation

$$\dot{\gamma}/\dot{\sigma} = N = K_{\rm m}/K_{\rm A} \tag{5}$$

Since there is negligible lag in the seeker and no coupling between the seeker and missile motions, the speed of response of the complete system depends predominantly on the dynamic characteristics of the missile-control-system combination.

METHOD OF ANALYSIS

The equations describing the dynamic characteristics of the missiles, control system, and seeker were solved for a step $\mathring{\sigma}$ input by means of a Reeves Electronic Analogue Computer for the rate feedback cases, and by means of the Ames High-Speed Electronic Simulator for cases where normal acceleration feedback was included. The $\mathring{\gamma}$ output responses were optimized for the design flight condition (M=2.7) by varying system gearings and the feedback lead terms to determine the most rapid response obtainable consistent with adequate stability.

For the low-speed condition (M=1.3) the optimized gearings and lead constants, as determined at M=2.7, were used with the changed aerodynamic parameters to determine the effect of the decreasing flight speed on



the responses. Comparison of $\mathring{\gamma}/\mathring{\sigma}$ steady-state values obtained by the electronic computers and those calculated using known gearings indicates that the REAC results are accurate to within 5 percent and the simulator results are accurate to within 10 percent.

In determining the effect of static margin, the parameter $X = x/d - C_{m_0^2}/\tau$ was varied because of the convenient manner in which it occurs in the missile transfer function (equations (3)). Results are presented with both the normal design value of X and with X = 0 for the control system having both rate and normal acceleration feedback. With rate feedback only, the results include just the normal design value of X.

The results are for a navigation ratio of 3. This value was chosen on a basis of desirable trajectory characteristics and anticipated noise effects. However, since for this guidance system the navigation ratio can be adjusted independently of the dynamics (see reference 2), the results apply for all navigation ratios that might be physically realizable.

RESULTS AND DISCUSSION

Rate Feedback Only

Normal static margin. The optimized responses of the three missiles at M=2.7 with the normal design static margin for the control system with rate feedback only are shown in figure 4(a). Corresponding values of the optimum gearings are listed-in table II. The variable-incidence configuration has only a slightly more rapid response than the other two configurations. It can be seen that this is due to the effect of the system lags on the initial lift build-up, as otherwise it would be expected that the variable-incidence missile would have a larger advantage in terms of speed of response. The oscillation frequency of all three responses is very nearly the missile design value (approximately 2 cycles per second).

In figure 4(b) are the responses for the same gearings as for figure 4(a) but with the M=1.3 aerodynamic parameters. The navigation ratio, N, has increased by about 30 percent for the variable-incidence and tail-control configurations and decreased by about 10 percent for the canard. This occurs because of the small optimum value of $K_{\rm S}K_{\rm C}K_{\rm C}^{\rm T}$ and the variation of the missile gearing, $K_{\rm C}^{\rm T}/{\rm X}$, with Mach number as is shown in the following:

$$N = K_{m}/K_{A} = (K_{S}K_{\theta}^{1}/X)/K_{A}(1 + K_{S}K_{G}K_{\theta}^{1}/X)$$

$$= K_{S}K_{\theta}^{1}/K_{A}(X + K_{S}K_{G}K_{\theta}^{1}) \approx (K_{S}/K_{A})(K_{\theta}^{1}/X)$$

$$K_{S}K_{G}K_{\theta}^{1} \ll X$$

$$(6)$$

since



For the variable-incidence and tail-control configurations, K_{θ}^*/X increases with decreasing Mach number while for the canard K_{θ}^*/X decreases.

Simplified trajectory studies show that in order to maintain desirable trajectory characteristics, a factor which depends on the navigation ratio, on the missile flight speed V, and on the missile-target closing speed V_C must be kept constant (reference 3). If this factor, which is denoted by n in this report (\equiv A in reference 3), is too large, trajectory instability results; if too small, a sluggish trajectory and large miss distance result. From the following definition of n

$$n = NV \cos (\sigma - \gamma) / [V \cos (\sigma - \gamma) - V_T \cos \sigma] \approx NV / V_C$$
 (7)

it can be determined that in order to maintain a constant n with decreasing missile flight speed, N must increase with decreasing flight speed for a head-on attack, remain constant for a beam attack, and decrease for a tail attack. It is apparent that without automatic gain adjustment, none of the missiles are capable of maintaining optimum trajectory characteristics with variations in flight speed for all initial conditions. However, the magnitude of the change in N for all three missile configurations appears to be small enough so that the above effect should not be serious for the speed variation considered herein.

Rate and Normal Acceleration Feedback

Design static margin. The optimized responses of the three missiles at M=2.7 with the normal design static margin for the control system with both rate and acceleration feedback are shown in figure 5(a). Corresponding values of the optimum gearings and lead constants are listed in table II. It is seen that, with normal acceleration feedback in the control system, the frequency of the over-all response, though different for the various missiles, is higher for all three missiles than the missile design frequency. This, of course, is the primary advantage of the acceleration control system: It is possible to attain an oscillation frequency, and therefore speed of response, higher than that of the airframe alone.

In terms of speed of response, the order of merit of the missiles is the reverse of that which occurred with rate feedback only, the tail control having the most rapid speed of response and highest oscillation frequency, but also having the largest initial overshoot. This is due to the relative magnitude of the numerator terms in the missile $\dot{\gamma}/\dot{\theta}$



transfer function, c_{γ_1} and c_{γ_2} . The small negative terms for the tail-control configuration result in a higher frequency than the small positive terms for the canard and result in a higher frequency than the larger positive terms for the variable-incidence configuration.

In figure 5(b) are the responses for the same gearings and lead constants as for figure 5(a) but with the M=1.3 aerodynamic parameters and missile flight speed. For all three missiles the navigation ratio has approximately doubled. This is due to the fact that the navigation ratio is approximately inversely proportional to the missile flight speed as shown below:

$$N = K_{m}/K_{A} = (K_{S}K_{\theta}^{*}/X)/K_{A}(1 + K_{S}K_{G}K_{\theta}^{*}/X + K_{S}K_{a}VK_{\theta}^{*}/X)$$

$$= K_{S}K_{\theta}^{*}/K_{A}(X + K_{S}K_{G}K_{\theta}^{*} + K_{S}K_{a}VK_{\theta}^{*})$$

$$\approx 1/K_{A}K_{a}V$$

$$X + K_{S}K_{G}K_{\theta}^{*} \ll K_{S}K_{a}VK_{\theta}^{*}$$

$$(8)$$

since

With large increases in the navigation ratio such as occur with this control system, trajectory instability is likely to occur for tail chases as the missile flight speed decreases unless some sort of automatic gain changer is provided.

Low static margin. The optimized responses at M=2.7 and the responses at M=1.3 are shown in figures 6(a) and 6(b), respectively, for X=0. At this static margin the missiles themselves are slightly unstable statically, but the optimized responses are almost identical to those for the normal design static margin. These results show the ability of an acceleration control system to provide a rapid response for a wide range of aerodynamic parameters. Since the responses are so nearly identical to those for the normal static margin, the previous discussion is equally applicable to the low-static-margin case.

CONCLUSIONS

The maximum speeds of response that could be attained by three missile configurations, a variable-incidence-wing, a canard, and a tail-control, in combination with a particular proportional navigation guidance system, were investigated. The missile control system utilized both rate-of-pitch feedback and normal acceleration feedback. From the results of the investigation, the following conclusions may be drawn:

- 1. The optimum configuration in terms of speed of response depends on the characteristics of the control system.
- 2. With rate-of-pitch feedback only, the oscillation frequency of the over-all response is very nearly that of the missile alone, and the variable-incidence configuration allows a slightly more rapid system response than either the canard or tail-control configuration.
- 3. With both rate-of-pitch and normal acceleration feedback, the oscillation frequency is higher than that of the missile alone, and the tail-control configuration allows the highest system oscillation frequency and the most rapid speed of response. However, this configuration also has the largest initial overshoot.
- 4. If normal acceleration feedback is used with a boost-glide missile, objectionable navigation ratio variations with missile-flight-speed changes may exist, necessitating automatic gain adjustment in flight.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif.

REFERENCES

- 1. Matthews, Howard F., and Stewart, Elwood C.: A Comparison of the Calculated Maximum-Maneuver Response Characteristics of Three Air-to-Air, Beam-Rider, Guided Missiles Having Different Lift Ratios. NACA RM A51F18, 1951.
- Abramovitz, Marvin: Theoretical Investigation of the Performance of Proportional Navigation Guidance Systems - Effect of Method of Positioning the Radar Antenna on the Speed of Response. NACA RM A52E27, 1952.
- 3. Anon: AN/DPN-7 Guidance System and Associated Equipment. Integrated Final Technical Report. Fairchild Rep. No. 6E-695, Feb. 19, 1951.
- 4. Johnson, R. L.: Sparrow Investigation of the Missile Minimum Natural Frequency Requirement. Douglas MTM 203, May 22, 1950.

TABLE I .- MASS AND AERODYNAMIC PARAMETERS OF MISSILES FOR 50,000 FEET ALITITUDE

		M=2.7		M≈1.3			
Parameter	Variable incidence	Canard	Tail control	Variable incidence	Canard	Tail control	
V, ft/sec m, slugs Iy, slug-ft ² Sf, ft ² Sr, ft ² If, ft (design) lr, ft (design) Sb, ft ² d, ft T, sec iy, sec ² (x/d),(design) (l/d) Cla, l/red Cla, l/red Cla, l/red Cmo, l/red/sec Ko, l/red/sec To, sec co, sec	2620 6.67 41.0 2.61 1.14 .20 4.56 .349 .667 40.4 .142 .990 1.96 23.8 9.87 0916 0218 .477 1.70 .866	2620 6.67 41.0 .86 2.97 3.69 3.22 .349 .667 40.4 .142 .990 9.89 23.8 2.93 151 0260 .716 1.70	2620 6.67 41.0 1.49 .90 .06 4.70 .349 .667 40.4 .142 1.37 -5.68 17.2 5.69 0780 0173 801 2.35 413	1260 6.67 41.0 2.61 1.14 .20 4.56 .349 .667 84.1 .615 .906 2.60 38.3 18.0 414 204 .556 2.20	1260 6.67 41.0 .86 2.97 3.69 3.22 .349 .667 84.1 .615 1.20 15.9 38.1 4.08 620 159 .772 2.19 .00129	1260 6.67 41.0 1.49 .90 .06 4.70 .349 .667 84.1 -5.64 26.3 16.6 -348 -1.12 3.19 -566	
c_{γ_2} , sec c_{γ_2} , sec c_{θ_1} , sec c_{θ_2} , sec	.00243 .00305 .00827 .00597	.000754 .000604 .0110 .00597	000974 00145 .00905 .00825	.00618 .0235 .0160	.00129 .00102 .0276 .0161		

TABLE II. - OPTIMIZED GEARINGS AND LEAD CONSTANTS

Configuration	Figure	Optimized gearings and lead constants							
		Km/KA	KSKaVK⊖ ⁴	ta	Kskgke •	^t G	Kakr		
Variable incidence Canard Tail	4(a) 4(a)	3	0	0 0	0.02	0.05	30 25		
control	4(a)	3	0	0	•015	•05	20		
Variable incidence Canard Tail	5(a) 5(a)	3	10 10	.013 .015		•05 •05	15 15		
control	5(a)	3	10	•005	•20	•06	15		
Variable incidence Canard Tail	6(a) 6(a)	3	10 10	.025 .015		.05 .05	15 15		
control	6(a)	3	10	•008	•24	•06	15		

Commence

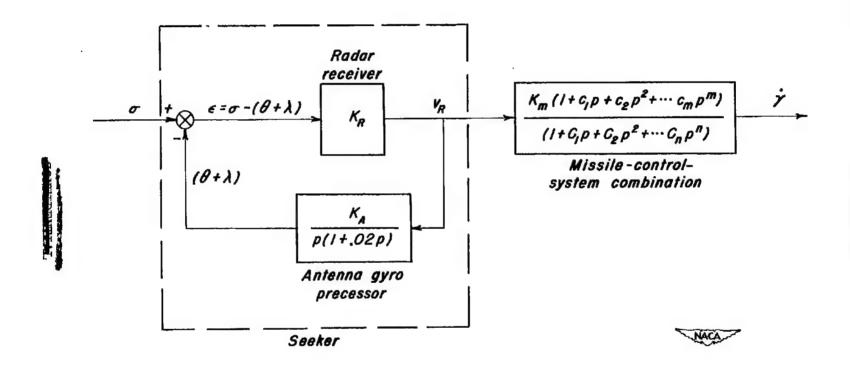
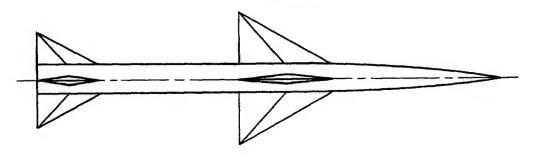


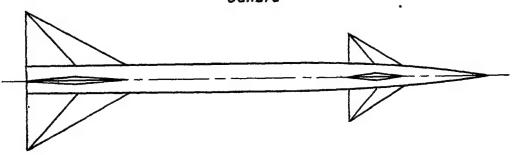
Figure 1.- Block diagram of proportional navigation guidance system.

Note: Rear surfaces rotated 45° to show relative areas.

Variable incidence



Canard



Tail control

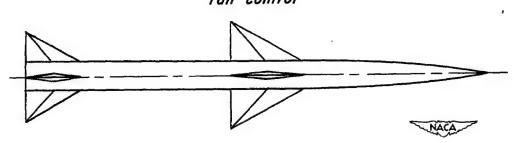


Figure 2.- Missile configurations.



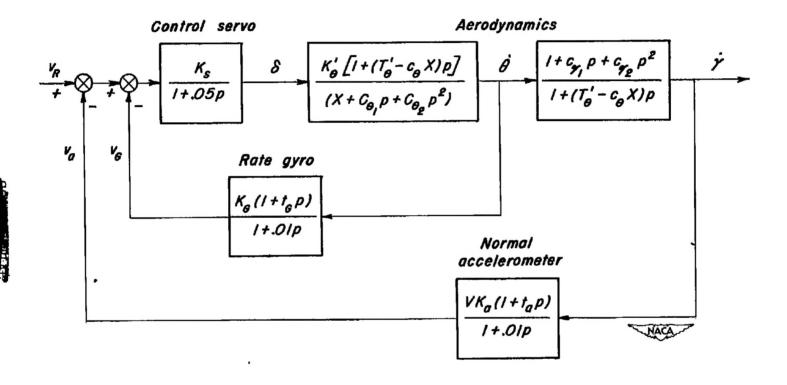


Figure 3.- Missile control system.

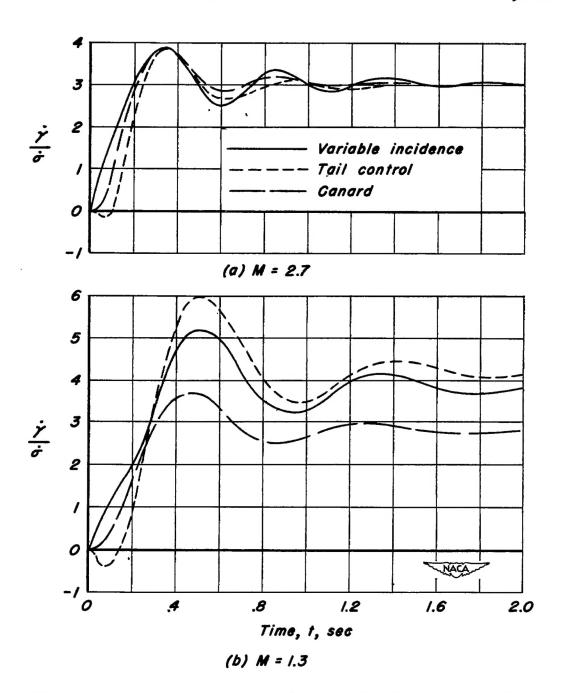


Figure 4.—Transient responses for rate feedback only in the control system with design static margin.

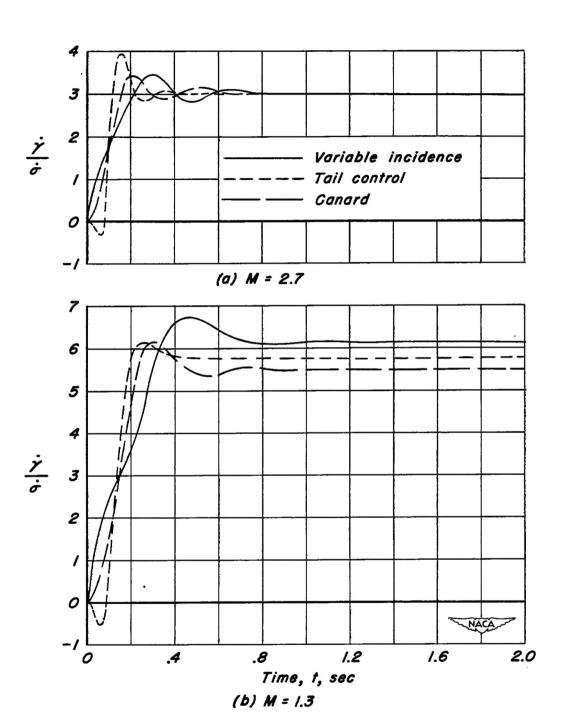


Figure 5.—Transient responses for normal—acceleration feedback in the control system with design static margin.

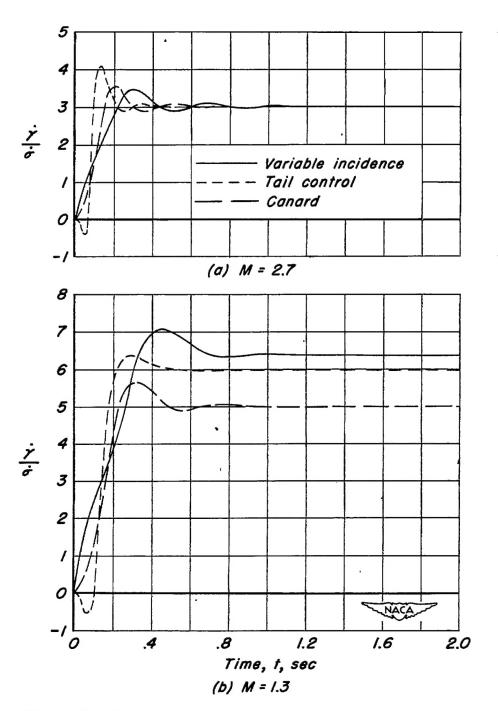


Figure 6.—Transient responses for normal-acceleration feedback in the control system with X = 0.

